Distinguishing between symplectic quotients by SU₂ using invariant theory

Christopher Seaton
Rhodes College
Department of Mathematics and Computer Science
seatonc@rhodes.edu

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Abstract

Let $K$ be (closed) subgroup of the group of $n \times n$ unitary matrices, considered as a group of linear transformations from $\mathbb{C}^n$ to itself. Forgetting the complex structure on $\mathbb{C}^n$, the underlying real vector space isomorphic to $\mathbb{R}^{2n}$ is an example of a symplectic manifold. The action of $K$ admits a moment map, which in this case is a collection of quadratic polynomials, and the symplectic quotient is defined to be $X = Z/K$ where $Z$ is the set of points on which the moment map vanishes. While $X$ is usually singular, it has several structures, including well-defined notions of what it means for a function $X \to \mathbb{R}$ to be smooth or polynomial. The algebra $\mathbb{R}[X]$ of polynomial functions on $X$ can be described using the real polynomial invariants associated to the group $K$.

We will describe methods from invariant theory that have been used to study the properties of a symplectic quotient $X$ via $\mathbb{R}[X]$. We will in particular focus on the case $K = SU_2$, the $2 \times 2$ unitary matrices with determinant 1, and discuss recent results regarding distinguishing between symplectic quotients.