Distinguishing between symplectic quotients by SU_2 using invariant theory

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Abstract

Let K be (closed) subgroup of the group of $n \times n$ unitary matrices, considered as a group of linear transformations from $\mathbb{C}^n$ to itself. Forgetting the complex structure on $\mathbb{C}^n$, the underlying real vector space isomorphic to $\mathbb{R}^{2n}$ is an example of a *symplectic manifold*. The action of K admits a *moment map*, which in this case is a collection of quadratic polynomials, and the *symplectic quotient* is defined to be $X = Z/K$ where Z is the set of points on which the moment map vanishes. While X is usually singular, it has several structures, including well-defined notions of what it means for a function $X \to \mathbb{R}$ to be smooth or polynomial. The algebra $\mathbb{R}[X]$ of polynomial functions on X can be described using the real polynomial invariants associated to the group K.

We will describe methods from invariant theory that have been used to study the properties of a symplectic quotient X via $\mathbb{R}[X]$. We will in particular focus on the case $K = \mathrm{SU}_2$, the 2×2 unitary matrices with determinant 1, and discuss recent results regarding distinguishing between symplectic quotients.